Probability

Ph.D. in Economics and Statistics, University of Milano-Bicocca
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PROGRAM.

- **Axioms of probability.** Definition of a probability space, the σ-additivity and its equivalent formulations. Elementary properties of probability measures and the inclusion-exclusion principle. The consequences of σ-additivity: continuity of probability measures, Boole's inequality. Some simple examples: the matching experiment.
- **Extensions of probability measures.** π-systems and λ-systems, the $\pi - \lambda$ Theorem. The Lebesgue measure on $(0, 1)$.
- **Denumerable probabilities.** Independence of events, elementary definition of conditional probability. Countable operations for almost sure and almost impossible events. The Borel-Cantelli lemmas and their applications. The Kolmogorov 0-1 law.
- **Random variables.** Definition of a random variable and its meaning. Expectation of a random variable: the general definition and its properties.
- **Convergences of random variables.** Almost sure convergence, in L^p and in probability. The relations among the different types of convergences. Counterexamples and partial converses.
- **Weak convergence.** The definition of weak convergence. Convergence in distribution of sequences of random variables. Equivalent characterizations of convergence in distribution. The relations between convergence in distribution and the other modes. The continuous mapping theorem for convergence in probability and in distribution. Slutsky's Theorem.
- **Characteristic functions.** The definition and its properties. The Lévy inversion theorem and the Lévy continuity theorem.
- **Limit theorems in probability: weak convergence.** The Lindeberg-Lévy theorem, Lindeberg and Lyapounov theorems. The delta method.
- **The strong law of large numbers.** Maximal inequalities, convergences of random series, the three series theorem. The Strong law of large numbers (Kolmogorov), the extension of Etemadi.

- **Conditional expectations.** The conditional expectation given a σ-field. Definitions and properties. Some applications to Statistics.

TEACHING METHODS. The teaching methods consist of traditional lessons and class exercises. The lectures require an interactive participation of the Ph.D. students, who will be asked, e.g., to solve exercises assigned during the course.

REFERENCES.

- Billingsley, P. (1995). *Probability and measure.* John Wiley & Sons, New York.
- Jacod, J. and Protter, P. (2004). *Probability essentials.* Springer, Berlin.
- Williams, D. (1999). *Probability with martingales.* Cambridge University Press, Cambridge.